MATH 201: Calculus and Analytic Geometry III

Fall 2016-2017, Final Exam, Duration: 2 hours

Problem	1	2	3	4	5	6	7	8	9	10	11	Total
Points	16	16	14	16	32	12	24	10	16	24	20	200
Scores												

Name:_________________________ **AUB ID:**________________________

Please circle your section:

Section 1
MWF 3, Nahlus
Recitation F. 11

Section 2
MWF 3, Nahlus
Recitation F. 10

Section 3
MWF 3, Nahlus
Recitation F. 8

Section 4
MWF 3, Nahlus
Recitation F. 9

Section 5
MWF 10, Shayya
Recitation T. 11

Section 6
MWF 10, Shayya
Recitation T. 12:30

Section 7
MWF 10, Shayya
Recitation T. 2

Section 8
MWF 10, Shayya
Recitation T. 5

Section 9
MWF 11, Yamani
Recitation F. 2

Section 10
MWF 11, Yamani
Recitation F 3

Section 11
MWF 11, Yamani
Recitation F. 4

Section 12
MWF 11, Yamani
Recitation F. 5

Section 13
MWF 2, Nahlus
Recitation M. 9

Section 14
MWF 2, Nahlus
Recitation M. 1

Section 15
MWF 2, Nahlus
Recitation M. 10

Section 16
MWF 2, Nahlus
Recitation M. 8

Section 17
MWF 9, Makdisi
Recitation Th. 9:30

Section 18
MWF 9, Makdisi
Recitation Th. 2

Section 19
MWF 9, Makdisi
Recitation Th. 8

Section 20
MWF 9, Makdisi
Recitation Th. 5

Section 21
MWF 1, Karam
Recitation F. 10

Section 22
MWF 1, Karam
Recitation F. 9

Section 23
MWF 1, Karam
Recitation F. 12

Section 24
MWF 1, Karam
Recitation F. 8

Section 25
MWF 10, Abi Khuzam
Recitation F. 4

Section 26
MWF 10, Abi Khuzam
Recitation F. 2

Section 27
MWF 10, Abi Khuzam
Recitation F 3

Section 28
MWF 10, Abi Khuzam
Recitation F. 1

Section 29
MWF 11, Aoun
Recitation Th. 3:30

Section 30
MWF 11, Aoun
Recitation Th. 2

Section 31
MWF 11, Aoun
Recitation Th. 5

Section 32
MWF 11, Aoun
Recitation Th. 12:30

INSTRUCTIONS:

(a) Explain your answers precisely and clearly to ensure full credit.
(b) Closed book. No notes. No calculators. No cellphones.
(c) UNLESS CLEARLY SPECIFIED OTHERWISE, THE BACKSIDE OF THE PAGES WILL NOT BE GRADED,

Problem 1

(16 pts) Find the tangent plane **and** normal line to the surface $x^2 + y^2 - z^2 = 6$ at the point (3,1,2).

Problem 2

(16 pts) Consider the region in the *xy*-plane given by $D = \{(x, y): x^2 + y^2 \le 5\}$
Find the extreme values (maximum and minimum values) of
$f(x, y) = x^2 + y^2 + 2x - y$ over *D*.

Problem 3

(14 pts) Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty}\left(\frac{1}{n} - sin\left(\frac{1}{n}\right)\right)$$

Problem 4

(16 pts) Find the Taylor series generated by $f(x)=\frac{1}{5x+6}$ at the point $x=-2$. Then find the largest open interval in which the series converges to the given function.
Hint: The resulting series is geometric.

Problem 5

Let R be the region in the **first quadrant** below the line $y = 1$ and above the parabola

$$y = \frac{x^2}{3}.$$

(a) (6 pts) SET UP BUT DO NOT EVALUATE $\iint_R dA(x,y) = \iint_R dA$ as an iterated double integral(s) in Cartesian coordinates using the order of integration $dydx$.

(b)(6 pts) SET UP BUT DO NOT EVALUATE $\iint_R dA(x,y) = \iint_R dA$ as an iterated double integral(s) in Cartesian coordinates using the order of integration $dxdy$.

(c) (12 pts) SET UP BUT DO NOT EVALUATE $\iint_R dA(x,y) = \iint_R dA$ as an iterated double integral(s) in polar coordinates using the order of integration $dr d\theta$.

(d) (8 pts) Evaluate $\iint_R dA(x,y) = \iint_R dA$.

Problem 6

(12 pts) Prove directly from the definition of differentiability that the function $f(x,y)=(1+x)(2+y)$ is differentiable at the point (0,0).

Problem 7

(24 pts) Find the center of mass of a solid of a constant density bounded from below by the cone $z = \sqrt{x^2 + y^2}$ and from above by the plane $z = 1$.

(You can continue your solution on the next page if needed.)

Problem 7 (Continue your solution here if needed)

Problem 8

(10 pts) Find the following limit using the sandwich theorem and ideas from the proof of the integral test.

$$\lim_{n \to \infty} \frac{1 + \frac{1}{2} + \ldots + \frac{1}{n}}{ln\left(n^3\right)}$$

Problem 9

(16 pts) Let R be the region in the xy-plane bounded by the lines $y = x,\ x - y = 4,\ x + y = 1$ and $x + y = 3$. Use the transformation

$x = \dfrac{u+v}{2}$, $y = \dfrac{v-u}{2}$ to rewrite $\iint\limits_R \dfrac{1}{x+y}\,dA(x,y) = \iint\limits_R \dfrac{1}{x+y}\,dA$ as an integral over an appropriate region G in the uv -plane. Then evaluate the uv integral over G.

Problem 10

Let D be the region bounded below by the plane $z = 0$, on the sides by the cylinder $x^2 + y^2 = 1$, and above by the sphere $x^2 + y^2 + z^2 = 4$.

(a) (7 pts) SET UP BUT DO NOT EVALUATE $\iiint_D dV(x, y, z) = \iiint_D dV$ as an iterated triple integral(s) in Cartesian coordinates using the order of integration $dzdydx$.

(b) (7 pts) SET UP BUT DO NOT EVALUATE $\iiint_D dV(x,y,z) = \iiint_D dV$ as an iterated triple integral(s) in cylindrical coordinates using the order of integration $dz\,dr\,d\theta$.

(c) (10 pts) SET UP BUT DO NOT EVALUATE $\iiint_D dV(x,y,z) = \iiint_D dV$ as an iterated triple integral(s) in spherical coordinates using the order of integration $d\rho\, d\phi\, d\theta$.

Problem 11

Suppose $g(u,v)$ and $h(u,v)$ are differentiable functions of two variables with continuous first order partial derivatives. Also suppose that

$$g(2,1) = 4 \quad \text{and} \quad \nabla g(2,1) = 2\mathbf{i} + 4\mathbf{j}$$

and

$$h(2,1) = 5 \quad \text{and} \quad \nabla h(2,1) = 4\mathbf{i} + 2\mathbf{j}.$$

(a) (10 pts) Give an approximate value of $g(2.2, 1.1)$ and $h(2.2, 1.1)$.

(b) (5 pts) Let C_1 and C_2 be the curves in the *xy*-plane given by

$$C_1 : \begin{cases} x = g(2, v) \\ y = h(2, v) \\ 1 \le v \le 1.1 \end{cases} \quad \text{and} \quad C_2 : \begin{cases} x = g(u, 1) \\ y = h(u, 1) \\ 2 \le u \le 2.2 \end{cases}$$

Approximate the lengths of C_1 and C_2.

(c) (5 pts) Let R be the image of the rectangle $2 \le u \le 2.2$, $1 \le v \le 1.1$ under the transformation

$$\begin{cases} x = g(u,v) \\ y = h(u,v) \end{cases}$$

Approximate the area of R.
Hint: R is approximately a parallelogram and its sides are not perpendicular, so its area is NOT approximately the product of the two lengths you found in part **(b)**.